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LAGRANGIAN TURBULENCE:
STRUCTURES AND MIXING IN ADMISSIBLE MODEL FLOWS

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May 8, 1987

ABSTRACT:
This report summarizes the main results obtained during the first two years of the grant AFOSR-87-0385. The general objective of our work during this period was to bridge the gap between modern ideas from dynamical systems and chaos and more traditional approaches to turbulence. In order to reach this objective we conducted theoretical and computational work on two systems: (i) a perturbed Kelvin cat eyes flow, and (ii) prototype solutions of the Navier-Stokes equations near solid walls. The main results obtained are two-fold: (a) we have been able to produce flows capable of producing complex distributions of vorticity, and (b) we have been able to construct flow fields, based on solutions of the Navier-Stokes equations, which are capable of displaying both Eulerian and Lagrangian turbulence.

INTRODUCTION

Recent developments in chaos theory and dynamical systems hold promise for the understanding of turbulence; however, the connection is by no means complete and the points of conflict appear not to have been clearly identified. *Chaos* admits various mathematical definitions (unfortunately, not all of them equivalent). In the context or volume or area preserving systems, chaos can be interpreted as: (i) the flow produces either transverse homoclinic or transverse heteroclinic intersections, (ii) the flow produces horseshoe maps; these definitions are amenable to mathematical proof. Oftentimes the definitions are computationally based; for example, in the context of dissipative systems 'chaos' can be interpreted as a system that has an attractor with at least one positive Lyapunov exponent, a so-called 'strange attractor'. Other diagnostics are somewhat less rigorous; for example, the visual appearance of numerically computed Poincaré sections. Alternative definitions, common in experimental studies, are broadband power spectrum of a signal obtained at a fixed point in the flow or a decaying correlation coefficient. A criticism of these diagnostics is that there is no spatial information regarding the complexity of the flow.

By contrast with chaos, *turbulence*, as understood by most workers in fluid mechanics as opposed to dynamical systems, is more akin to an illness typified by a number of symptoms (i.e., a syndrome). A flow is diagnosed to be turbulent if it presents typical symptoms [though by no means will all researchers agree as to which ones are the 'right symptoms']. Typical symptoms are: (i) the Eulerian signal of quantities such as velocity and pressure are complicated (temporal disorder), (ii) the flow has ability to mix, (iii) there is energy transfer from large to small scales--characterized by spectral laws and regimes, (iv) there is mixing of vorticity (spatial disorder), (v) there is vorticity intensification until intensification is balanced by the dissipation, (vi) the flow is characterized by a large Reynolds number[1]. It is clear that there are chaotic flows which are not turbulent. As to whether turbulent flows are chaotic the answer depends largely upon the definition adopted. Chaotic advection clearly mimics (ii). However, every flow based on perturbing an analytical expression for the streamfunction (or velocity field) cannot possibly create a complex pattern of vorticity since at the most the vorticity will as complicated as the velocity field itself ($\omega=\nabla \times v$). Another difficulty is that every flow based on perturbing an analytical expression for the streamfunction (or velocity field) cannot create Eulerian turbulence (the velocity at a fixed point is simply given by the steady flow plus the perturbation which is given *a priori*). Can these difficulties be overcome? What kinds of things can (and cannot) be understood in terms of the chaotic advection approach?

ADVECTION OF VORTICITY:

The best studied flows examined in the context of chaotic advection (and the only ones for which there are reliable experiments) are *2d*-time periodic Stokes flows.[2,3,4] (i.e., Reynolds$\rightarrow$0 and Strouhal$\rightarrow$0). In this case, the streamfunction adjusts instantly to time-dependent boundary conditions, and even though a passive scalar might be mixed chaotically by the flow, $\psi(x,t)$ is never truly complex. In particular, the vorticity distribution *is not* advected by the flow and satisfies $\nabla^4\psi = -\nabla^2\omega_z = 0$. The situation is obviously different at finite and large Reynolds numbers. In *3d* and in the limit Re$\rightarrow\infty$, the vorticity equation reduces to

$$D\omega/Dt = \omega.\nabla v\,, \qquad (1)$$

[1]see for example H. Tennekes and J.L. Lumley A First Course in Turbulence. 6th printing, Cambridge: MIT Press (1980).

[2]J.M. Ottino, C.W. Leong, H. Rising and P.D. Swanson, Nature. 333, 419 (1988).

[3]J. Chaiken, R. Chevray, M. Tabor, and Q.M. Tan, Proc. Roy. Soc. London. A408, 165 (1986).

[4]J.M. Ottino, Scient. Amer., 260, 56 (1989).

vortex lines move as material lines, and both can be stretched and folded into complex structures characteristic of chaotic flows. The solution of (1), $\omega=\omega_0.F^T$, where ω_0 is the initial value of the vorticity, and **F** the deformation tensor, indicates the very same same point. This result might give also the mistaken impression that the evolution of vorticity can be calculated on purely kinematical grounds. However, this is not true; the deformation tensor **F** cannot be calculated until the velocity field is obtained by solving Euler's equation.

All the three-dimensional flows studied to date in the context of chaotic advection are unable to shed much light into the connection between chaos and vortex stretching. In particular if **v** is *given*, as for example in the ABC flow[1,2,3], the vorticity is simply given by $\omega=\nabla\times\mathbf{v}$ and there is nothing else left to do. On the other hand, if the flow is based on singularities[4] the vorticity is uniform everywhere (generally zero, except at the vortices themselves) and there is no mixing of vorticity to speak of even in the case of perturbed flows. However, what happens in perturbed flows with a *distribution* of vorticity? (i.e., $\nabla\omega\neq0$). We decided to tackle this question in terms of the Kelvin cat eyes flow which is an exact solution of Euler's equation[5] (for a somewhat more classical analysis, see[6]).

KELVIN CAT EYES FLOW

The streamfunction with respect to a fixed laboratory frame (x',y') is of the form

$$\psi'(x',y',t) = uy' + \ln\,[\cosh(y') + A\cos(x' - ut)] \quad . \qquad (2)$$

This flow presents a succession of hyperbolic and elliptic points with connecting heteroclinic orbits and produces chaos under a time-perturbation $v_x=\varepsilon\sin(\omega t)$ [u represents the average speed of vortices moving from left to right and A is a parameter quantifying the concentration of vorticity, A=1 corresponds to point vortices; in this case the vorticity is zero everywhere except at the point vortices themselves]. The flow stretches and folds material lines. Computational studies, e.g., Poincaré sections, behavior of manifolds, as well as analytical techniques

[1] T. Dombre, U. Frisch, J.M. Greene, M. Hénon, A. Mehr and A.M. Soward. J. Fluid Mech. **167**, 353 (1986).

[2] M. Feingold, L.P. Kadanoff and O. Piro, O. J. Sat. Phys. **50**, 529 (1988).

[3] V.V. Beloshapkin, A.A. Chernikov, M. Ya. Natenzon, B.A. Petrovichev, R.Z. Sagdeev and G.M. Zaslavsky. Nature, **337**, 133 (1989).

[4] e.g., blinking vortex flow, H. Aref, J. Fluid Mech., **143**, 1, 1984, oscillating pair of vortices (V. Rom-Kedar, A. Leonard, and S. Wiggins, J. Fluid. Mech., submitted 1989).

[5] J.T. Stuart, J. Fluid Mech. **29**, 417 (1967).

[6] G.M. Corcos and F.S. Sherman, J. Fluid Mech., 139, **29** (1984).

(Melnikov method) show chaotic behavior [we note, in passing, that for a fixed ε, the chaotic behavior is maximized for an intermediate value of the frequency ω]. Since the fluid is inviscid it has to satisfy Euler's equation

$$D\omega_z/Dt = (\partial\omega_z/\partial t)_{X,Y} = 0\,, \tag{3}$$

and isovorticity lines have to move as material lines. However, it is relatively easy to show that the perturbed cat eye flow no longer satisfies the Euler equation, apart from three exceptional cases: (i) ε=0, perturbation strength is zero (no chaos); (ii) A=0, hyperbolic tangent profile (no saddle connections and therefore no chaos); and (iii) A=1, point vortices, saddle connections (chaos, but no mixing of vorticity since ω_z is identically zero).

One possibility is to regard the parameter A in equation (10 as a new parameter with initial value A_0 and to force each particle in the flow to conserve its own vorticity, i.e., $(D\omega_z/Dt)_{A0}$=0. The most elucidating simulation is to follow the evolution of an iso-vorticity contour (Figure 1). It is evident that the vorticity is stretched and folded and regions of high vorticity can come in close contact with regions of different vorticity. By contrast, the flow without the vorticity constraint distorts material lines but the iso-vorticity lines do not change at all; in fact, the isovorticity contour is not even affected by the time-perturbation! It does therefore appear that if want to stay within the confines of chaotic advection we have three choices: (i) ignore Euler's equation (no mixing of vorticity), (ii) ignore Euler's but advect fluid particles conserving their initial vorticity, and (iii) somehow bring up Euler's equation into the picture, as advocated here. The most 'realistic case" is (iii). However, we have found close agreement between methods (ii) and (iii). This result is important and suggests that as a first approximation *it might be possible to advect vorticity as a purely passive scalar.*

FLOWS NEAR WALLS:

To investigate the possibility of generating Eulerian turbulence we have considered flows which are asymptotically exact solutions of the Navier-Stokes and continuity equations in two and three dimensions. The starting point is to expand the Eulerian velocity field in a Taylor series around a point **p**, i.e.,

$$\mathbf{v(x)} = \mathbf{v(p)} + \mathbf{(x\text{-}p).\nabla v(x)}|_{\mathbf{x=p}} + 1/2\,\mathbf{(x\text{-}p)(x\text{-}p)\!:\!\nabla\nabla v(x)}|_{\mathbf{x=p}} + \dots \tag{4}$$

Each term $(1/n!)\nabla^n v(x)|_{x=p}$ in the expansion represents a tensor of order n+1. Denoting $A_{ijk...}=(1/n!)\partial^n v_i/\partial x_j \partial x_k ...$, the expansion can be written as

$$v_i = A_i + A_{ij}\, x_j + A_{ijk}\, x_j x_k + A_{ijkl}\, x_j x_k x_l + A_{ijklm}\, x_j x_k x_l\, x_m + ... \tag{5}$$

The tensors $A_{ijk...}$ constitute the unknowns and are to be found by forcing the series to satisfy the continuity and Navier-Stokes equations as well as the boundary conditions of the problem in question. Substituting the velocity expansion into the continuity and Navier-Stokes equations and equating coefficients of equal power generates a series of independent relationships between the coefficients $A_{ijk...}$. The coefficients are then forced to satisfy "boundary conditions", such non-slip, impenetrability of the wall, and specification of surface vorticity (a way to impose separation and reattachment points, such as in a separation bubble; see Figure 2(a-c)), etc. We investigated several types of flows as well as several time dependent pertubations and computed the time evolution of the coefficients $A_{ijk...}$, which in turn control the Eulerian velocity field. We have discovered that the evolution equations for the coefficients can display strange attractors indicating Eulerian turbulence in the velocity field (see Figure 2(d)). To our knowledge, this is the first example of a flow, in the context of chaotic advection, possessing a strange attractor.

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J.M. Ottino, "***The Kinematics of Mixing: Stretching, Chaos, and Transport***" Cambridge University Press, 1989 (primarily last sections of Chapter 8).

J. M. Ottino, C.W. Leong, H.Rising, and P.D. Swanson, "Morphological Structures Produced by Mixing in Chaotic Flows", **Nature, 333**, 419-425, 1988 (cover article).

J.M. Ottino, "The Mixing of Fluids", **Scientific American, 260**, 56-67, 1989 (cover article).

T.J. Danielson and J.M. Ottino, "Chaos and Turbulence in Terms of Simple Flows", paper DH-10, **41st Meeting of the Division of Fluid Dynamics**, American Physical Society, Buffalo, New York, November 20-22, 1988.

J.M. Ottino, "Chaotic Advection of Fluids" commissioned article to appear in Vol. 22 of **Annual Reviews of Fluid Mechanics.**

T.J. Danielson and J.M. Ottino, "Mixing of Vorticity: Some Difficulties Connecting Chaotic Advection and Turbulence", to be submitted to **Phys. Fluids A.**

T.J. Danielson and J.M. Ottino, "An Examination of Chaos and Turbulence in Model Flows Near Walls", in preparation.

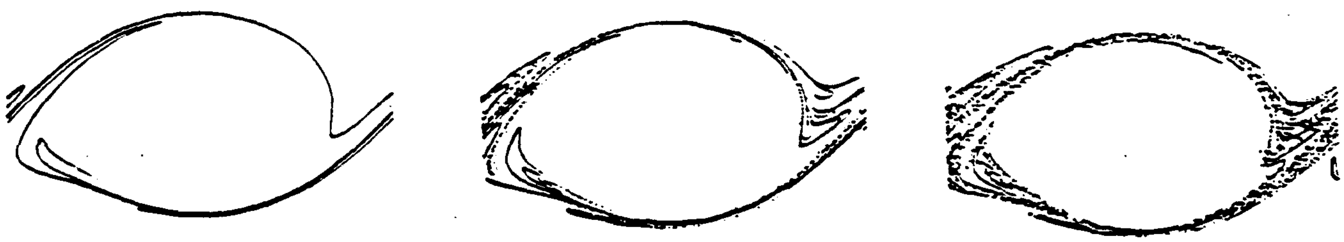

Figure 1: Time evolution of an iso-vorticity contour in the Kelvin cat eye flow.

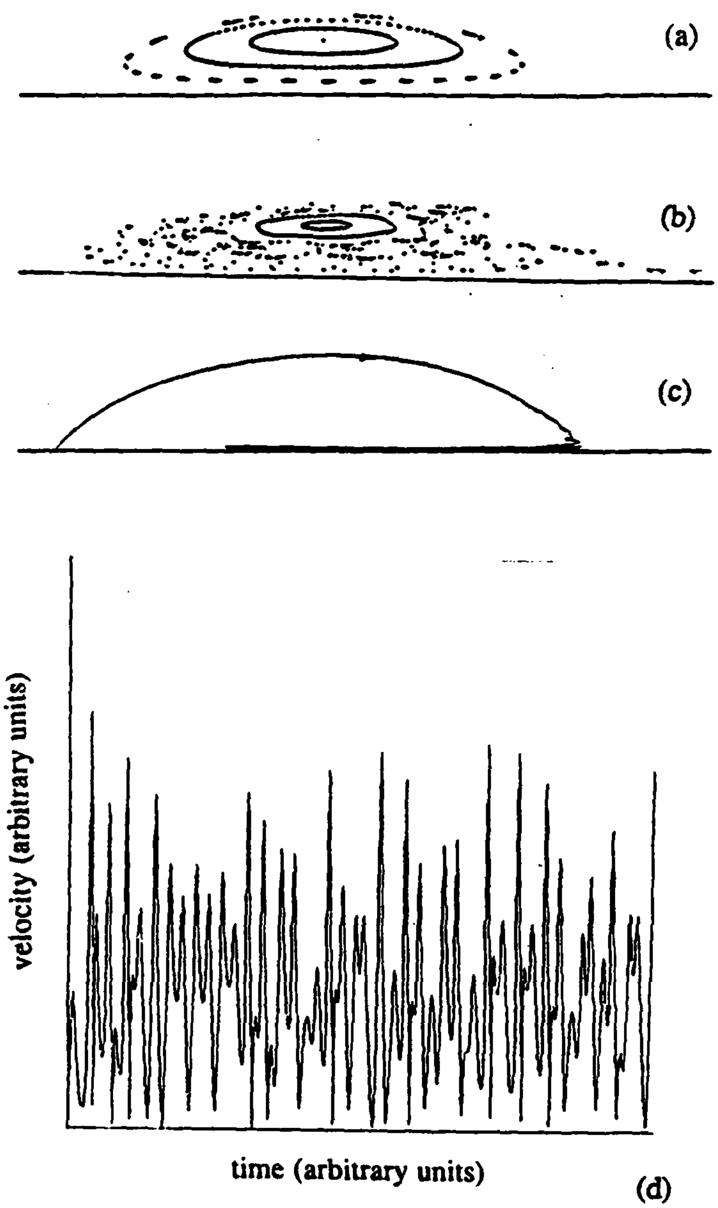

Figure 2: Typical behavior of a separation bubble under a time-periodic perturbation of the elliptic point. (a) Poincaré section for the integrable system; (b) Poincaré section for the perturbed system; (c) behavior of the unstable manifold; note the leaking as the manifold tries to reattach to the wall. Figure (d) shows the Eulerian velocity signal at a point located at a fixed position within the bubble.